Study of Fluctuations in an Arc Plasmajet using Electromagnetic Induction

Allen E. Fuhs*
Aerospace Corporation, El Segundo, Calif.

Interaction of a fluctuating conducting gas with a steady applied magnetic field drives fluctuating currents. The unsteady magnetic field arising from these currents induces a voltage V in a sensing coil. An integral equation for V and the related partial differential equation for the electrical potential ϕ are derived. A relation between signal rms magnitude and jet properties is discussed. Experiments were conducted using an arc plasmajet operating with argon. Most of the signal was due to frequency components in the interval 10 to 100 kc. By use of two transducers, one located downstream of the other, the average jet velocity was measured. The delay giving the peak value of the cross-correlation function formed from the two signals was taken as the transit time. Methods for electronically determining the time delay are described.

I. Introduction

INSTRUMENTS have been developed which measure the product of electrical conductivity and velocity, σU.[1, 2] Other instruments for measuring the profile of σU have been developed.[3, 4] To determine the value of σ from a determination of σU it is necessary to know U. The velocity can be estimated, or it can be measured separately. One motivation for the experiments described herein was to measure, with one transducer, both the product σU and U. If a fluid has a characteristic feature that is convected, then a measurement of the time for transit over a known distance gives an average velocity. Blackman[5] observed that the intensity of visible radiation fluctuates and that these changes in intensity are convected. The average velocity was measured using two photomultiplier tubes. Variation in light intensity is a characteristic quantity that is convected; another convected quantity is variations in σU. These changes in σU can be detected by means of the currents driven by the motion of a conductor in an applied magnetic field.

Turbulence in fluids has been studied by Kolin,[6] Grossman and Charwat,[7] and Grossman, Li, and Einstein[8] using electromagnetic induction. These investigators used electrodes and measured the potential difference due to the interaction of a moving fluid with an applied magnetic field. The electrical conductivity σ of the fluids (e.g., water) was small so that currents were correspondingly small. The potential difference under these conditions is a direct measure of the local velocity.

For the case of an arc plasmajet, σ is sufficiently large so that currents are induced and the simple relation between potential difference and local velocity is no longer valid. Although the relation is no longer valid, the currents permit detection of fluctuations in σ, U, or in σU without use of electrodes. The unsteady magnetic field arising from these currents induces a voltage V in a sensing coil. The second motivation for conducting the experiments was to determine whether or not fluctuations could be detected. Section II develops the theory. Section III describes the apparatus and reports the results, and Sec. IV discusses and interprets the results. Section V states the conclusions.

Presented as Preprint 64-375 at the AIAA 1st Annual Meeting, Washington, D. C., June 29–July 2, 1964; revision received November 12, 1964. The experiments originated as a result of discussions with R. Betchov, whose help and interest are appreciated. M. H. Dazey set up the experiment in which the time delay was registered by a marker pip. O. L. Gibb assembled the transducer and participated in the experiments. The experiments were conducted using the arcjet facilities at Space Sciences Division of Norair, Hawthorne, Calif. The cooperation of Sterge Demetriades is appreciated.

* Staff Scientist, Plasma Research Laboratory; now Visiting Fellow, Joint Institute for Laboratory Astrophysics, Univ. of Colorado, Boulder, Colo.; also Visiting Associate Professor, Aerospace Engineering Sciences Department, University of Colorado, Boulder, Colo. Associate Fellow Member AIAA.

II. Theoretical Background

A. Formula for the Sensing Coil Voltage

Consider the coil geometry illustrated in Fig. 1. The voltage induced in the sensing coil is

$$V = -N \frac{\partial}{\partial t} \int_A \mathbf{B}_i \cdot \mathbf{n} \, dA \tag{1}$$

where A is the cross-sectional area of the sensing coil with normal $\mathbf{n}$ and with N turns. The induced magnetic field, $\mathbf{B}_i$ can be divided into steady and time-varying parts. The steady magnetic field obviously does not give a signal. The nonsteady $\mathbf{B}_i$ is due to currents in the plasma which are given by

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{U} \times \mathbf{B}) \tag{2}$$

where σ is the d.c. electrical conductivity (scalar), $\mathbf{E}$ is the electrical field, $\mathbf{U}$ is the instantaneous flow velocity, and $\mathbf{B}$ is

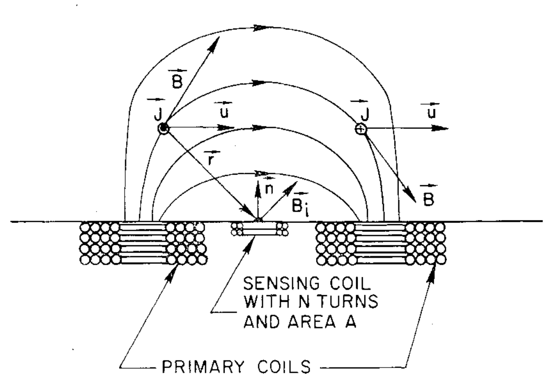

Fig. 1 Transducer coil arrangement. An electromagnetic field is induced in the sensing coil by the time rate of change of normal component of B_i.

the applied field. The applied field **B** must be small enough so that the ratio of electron cyclotron frequency to electron-neutral collision frequency is much less than unity. For most applications involving a partially ionized gas, e.g., re-entry plasma sheath, MHD generators, and arc plasmajets, the magnetic Reynolds number $\sigma\mu UL$ is much less than unity. The magnitude and spatial dependence of the applied field is assumed to be known. An electric field **E** may result from time-varying magnetic fields or from charge separation. There is no **E** due to the applied **B** since it is steady. For this case, **E** due to charge separation is not zero. The appropriate differential equation for **E** is derived in the following section.

The magnetic field linking the sensing coil is

$$\mathbf{B}_i = \frac{\mu}{4\pi}\int_V \frac{\mathbf{J}\times\mathbf{r}}{r^3}\,dV \tag{3}$$

Combining Eqs. (1–3) yields

$$V = -\frac{NA\mu}{4\pi}\frac{\partial}{\partial t}\int_V \frac{\sigma(\mathbf{E}+\mathbf{U}\times\mathbf{B})\times\mathbf{r}}{r^3}\cdot\mathbf{n}\,dV \tag{4}$$

In Eq. (4) it has been assumed that A is small so that $\mathbf{B}_i$ is constant over A. Integration over dA has been replaced by A. The time-dependent quantities in the preceding equation are $\mathbf{E}$, σ, and $\mathbf{U}$. The quantity $\mathbf{E}$ is time-dependent, as we shall see, as a result of Eq. (5) relating the potential to σ and $\mathbf{U}$.

B. Equation for the Electric Field

Since $\nabla\times\mathbf{E}$ is zero, the electric field can be expressed as the gradient of an electrostatic potential. Displacement currents are neglected relative to conduction currents. Taking the divergence of Ohm's law, Eq. (2), and expressing $\mathbf{E}$ as $-\nabla\phi$ yields

$$\sigma\nabla^2\phi + \nabla\sigma\cdot\nabla\phi = \nabla\cdot\sigma(\mathbf{U}\times\mathbf{B}) \tag{5}$$

Interpretation of the signal V in terms of the fluctuations in σU involves a simultaneous solution of Eqs. (4) and (5). The applied magnetic field **B** and the signal V are known.

C. Estimation of Signal Magnitude

In view of the complexity of the coupled equations for σU, an order of magnitude estimate of V is desirable. By assuming the induced current flows in a loop of radius r and cross-sectional area A, the induced magnetic field can be estimated. For an arcjet, r will be some fraction of the jet radius. For a turbulent boundary layer of a plasma sheath, r will be less than the boundary-layer thickness. The induced magnetic field is $B_i = 2\pi\sigma U'BA/10r^7 = (2\pi)(1)(100)(0.01)(10^{-7})/(10^4)(0.01)$. Typical values (MKS units) are used. The voltage induced in a sensing coil with $N = 1000$ turns would be $\omega B_iAN = (10^5)(10^{-8})(10^{-4})(10^3) = 10^{-4}$ v. Certainly 100 μv can be amplified with ease. There is a question as to relative strength of stray magnetic fields due to other causes.

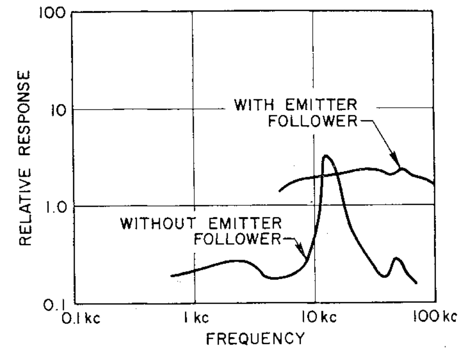

Fig. 3 Frequency response of the turbulence transducer.

III. Apparatus and Typical Results

An experiment was performed using the coil arrangement shown in Fig. 2 and an arc plasmajet. The primary coils that are mounted on the outer legs of E laminations were excited d.c. and had alternating poles. The magnetic field was 100 gauss at the pole face decreasing to 16 gauss at 0.5 in. and 6 gauss at 1 in. Emitter followers were used to isolate the sensing coils, mounted on the middle leg of the E lamination, from the distributed capacitance in the external leads. The frequency response with and without emitter followers is shown in Fig. 3. The response curves were obtained by placing a coil with 100 turns of wire above the sensing coil. The coil was driven with a constant voltage source, and the voltage induced in the sensing coil was measured as a function of frequency.

A schematic of the circuit is shown in Fig. 4. The four primary coils were wired in series so that the current in each was identical.

The jet was argon. Using other experimental techniques, the velocity of jet was measured; the method described in Sec. IV gave values within 20%. The conductivity was calculated assuming thermodynamic equilibrium and isentropic expansion in the nozzle. An energy balance was used to

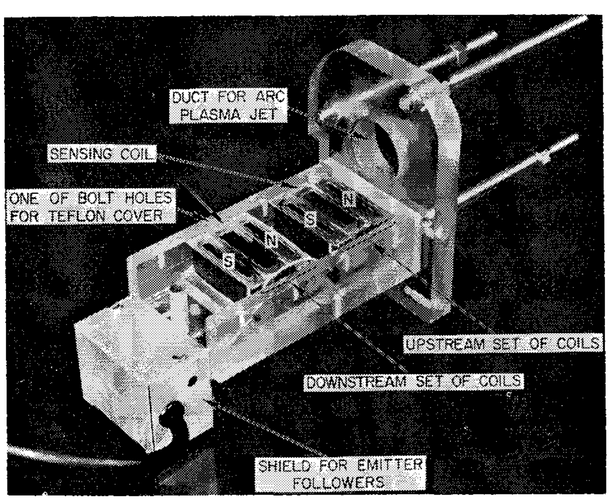

Fig. 2 Velocity and turbulence transducer.

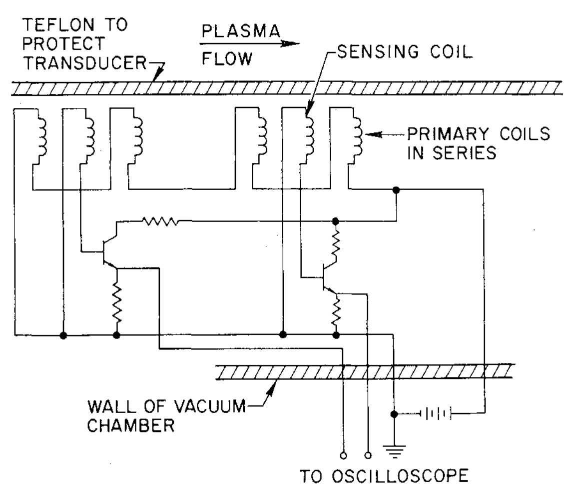

Fig. 4 Voltages induced in the sensing coils by the plasma flow recorded by photographs of oscilloscope traces.

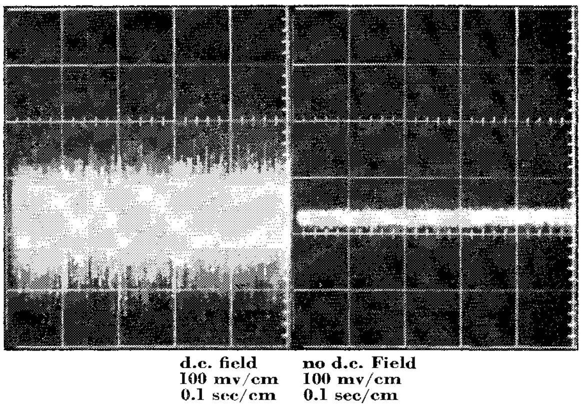

Fig. 5 Signal level with and without applied d.c. field. Arcjet was operating for each case.

obtain the stagnation enthalpy. Arc chamber pressure and the ambient pressure within the vacuum chamber were measured. Based on the calculations, σ was about 1000 mho/m. Based on measurements similar to those reported in Ref. 1, the value of σ was estimated to be approximately 2000 mho/m. The jet exhausted into a large chamber in which the ambient pressure was 2 to 5 mm Hg. The nozzle exit had a diameter of 1 in.

Voltage induced in the sensing coil while the arcjet was in operation with and without the applied d.c. was measured. Results are shown in Fig. 5. The signal due to the interaction of the applied d.c. field and the plasma flow is about 20 times greater than the signal due to unsteady magnetic fields from the arcjet. This test indicates that the signal is, in fact, the result of an interaction between the applied magnetic field and the plasma flow.

The output from the two sensing coils was examined using a dual-beam oscilloscope (either a Tektronix Model 551 or 555). Typical oscillograms obtained by using the lower beam to record the signal from the upstream coil set and the upper beam for the downstream set are shown in Fig. 6. Results are discussed in Sec. IV.

Another method of recording the data was to set the oscilloscope to trigger when the signal from the upstream coil set had a definite voltage and a positive slope. Time exposures were taken. A typical oscillogram is shown in Fig. 7. For run 2 the time delay was 22 μsec, which agrees well with the oscillograms appearing in Fig. 6.

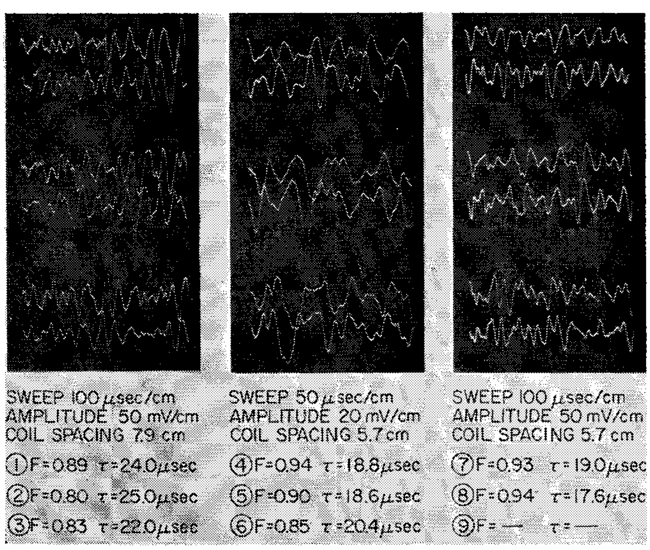

Fig. 6. Typical oscillograms.

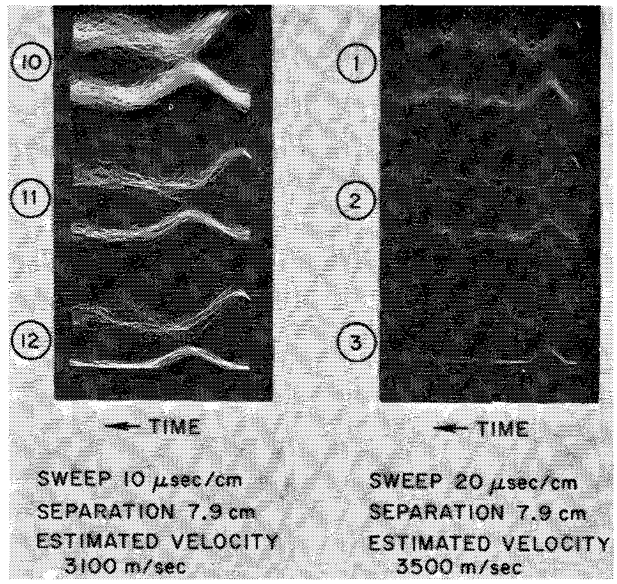

Fig. 7 Time exposure of multiple sweeps triggered by signal from upstream coil set.

Another method† uses the separate time bases of a Tektronix Model 555. The signal from the upstream coil set V_1 triggers time base A and starts both beams sweeping at 5 μsec/cm. V_1 is displayed on the upper trace. The signal from the downstream coil set V_2 triggers time base B that is set to sweep at 0.5 μsec/cm. The voltage for the horizontal deflection plates, which is obtained from time base B, is displayed on the lower beam. V_2 is not displayed but serves only to trigger time base B. Both A and B are triggered at the same voltage and same sign of slope of V. Typical oscillograms are shown in Fig. 8. Most of the pips, which indicate when V_2 triggered time base B, are confined to a narrow time band. There are scattered pips.

Another experiment using wire loops was performed to determine the response to different scale disturbances. The

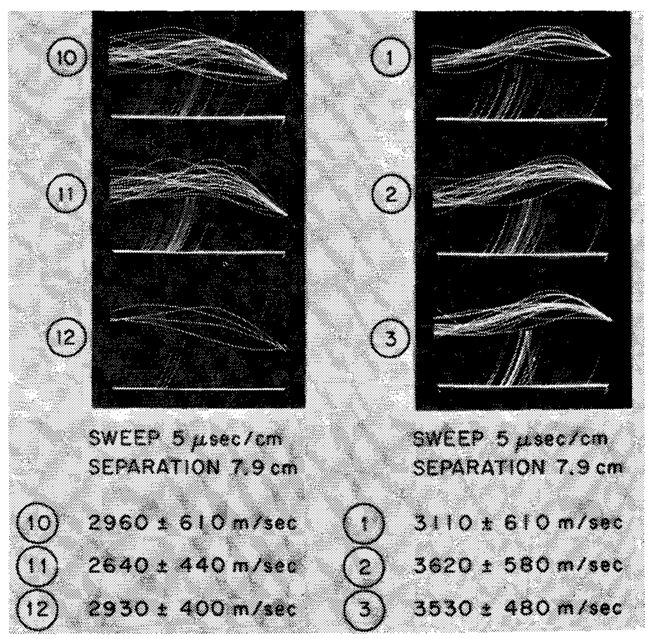

Fig. 8 Time delay given by position of marker pip.

† This method was set up by Mitchell Dazey of the Plasma Research Laboratory. His help and suggestions on experimental technique are appreciated.

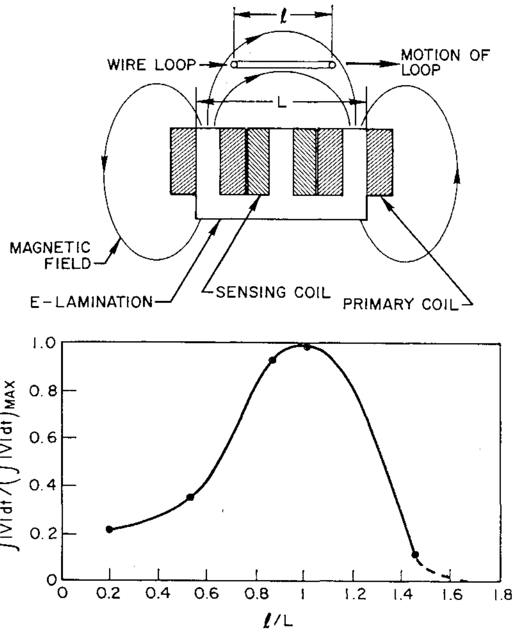

Fig. 9 Response to different scale disturbances. *V* is the voltage induced in the sensing coil.

loops were mounted on a large turntable, described in Ref. 2, and were moved through the magnetic field of the transducer. The integral of the absolute voltage is compared for different scale loops. Figure 9 shows schematically the experimental arrangement and the relative response to different size loops. Loops of the same scale as the transducer give maximum response.

IV. Discussion of Results

Three methods for determining the transit time have been presented in Sec. III. These were single sweep with slow sweep rate as shown in Fig. 6, the multiple exposure method illustrated by Fig. 7, and the pip marker method as shown by Fig. 8. Numbers identify different runs; runs with the same number in Figs. 6–8 were obtained under the same conditions.

The cross-correlation function

$$F(\tau) = \frac{\int V_1(t) V_2(t + \tau)\, dt}{(\int V_1^2\, dt)^{1/2} (\int V_2^2\, dt)^{1/2}} \tag{6}$$

was calculated for the oscillograms shown in Fig. 6. Both the peak value of F and the time delay for maximum F appear in Fig. 6. Values for F ranged from 0.80 to 0.94; a value of unity for F means perfect correlation. The disturbances are convected with a slight change in structure, which can also be seen in Fig. 7. If the signal from the downstream transducer were not highly correlated with the upstream signal, there would not be the well-defined maximum in the lower trace. The transit time, as determined by the three methods, has a statistical distribution; for example, runs 1–3 in Fig. 6 were obtained at the same arcjet conditions. The value of τ has scatter, which is more apparent in Fig. 8. A velocity was determined for each marker pip, and the average was calculated. The standard deviation

$$s = \left[\frac{1}{n} \sum_{i=1}^{n} (U_i - \bar{U})^2\right]^{1/2} \tag{7}$$

is given as the ± value. Run 10 had an average velocity of 2960 m/sec with s = 610 m/sec. Velocity measurements are summarized in Fig. 10 as a function of arcjet power and argon enthalpy. Runs 1–3 were at 10 kw, runs 4–9 were at 11.7 kw, and runs 10–12 were with arcjet power of about 15.3 kw. The dashed line represents the trend in the increase of velocity with argon enthalpy.

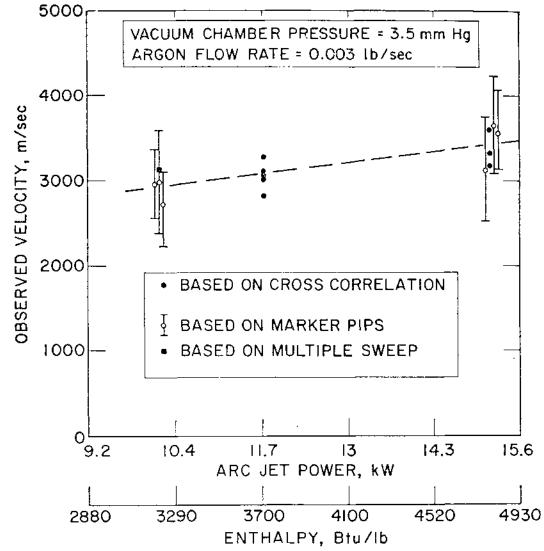

Fig. 10 Measured velocity as a function of argon enthalpy and arcjet power.

V. Conclusions

Currents driven by fluctuations in σU, as they exist in an arcjet, can be detected. For the conditions of the tests, the signal was about 50 mv rms in the frequency interval 10 to 100 kc. Signal-to-background-noise ratio was about 20.

The transit time for determining the average jet velocity was measured by means of the cross-correlation function using oscillograms as shown in Fig. 6, the delay in a peak value as shown in Fig. 7, and the pip marker method illustrated by Fig. 8. All three methods gave consistent results with an accuracy of at least 20%, sometimes 10%.

The theory is in a primitive state of development. The simultaneous solution of Eqs. (4) and (5) is a formidable task.

The fluctuations in σU are convected with little change in a time interval of the order of 20 μsec. This is evidenced by the large values of the cross-correlation function F, as given by Eq. (6). Besides the utilitarian value as a marker for a velocity transducer, the signal spectrum is related to the spectrum of turbulent fluctuations.

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